Kink production in the presence of impurities

T. Dobrowolski
Institute of Physics AP, Podchorążych 2, 30-084 Cracow, Poland
(Received 31 October 2001; published 5 March 2002)

The production of kinks during a quench in an overdamped regime of ϕ^4 model is investigated. Expelling defects from regions of nonzero force coming from the impurity are predicted.

DOI: 10.1103/PhysRevE.65.036136 PACS number(s): 64.60.−i, 05.45.−a, 47.54.+r, 47.20.Ky

I. CONTEXT

The kink-bearing ϕ^4 model is very popular because it has properties representative of those found in many applications in condensed matter physics [1], nuclear physics [2], and biology [3]. The process of the formation of kinks is the most interesting aspect of their evolution. The density of kinks is associated with the dynamics of the order parameter. As a consequence of the critical slowing down, the correlation length diverges; perturbations of the order parameter take longer to propagate over correlated regions, and therefore it takes longer to reach equilibrium. When the time remaining before the transition equals the equilibrium relaxation time, the correlation length can no longer adjust quickly enough to follow the changing temperature or the pressure of the system. The same time after a quench the system regains capacity to respond for changes of external parameters. The correlation length at that time (freeze-out time) sets the characteristic length scale for the initial kink network [4]. Until now this general picture has been verified in many physical and biological contexts.

In spite of the fact that matter is generically populated by impurities, all the results obtained hitherto concern homogenous and isotropic medium. As is well known, the presence of impurities and admixtures may completely change properties of the system. For instance, magnetic impurities break time-reversal invariance and therefore destroy superconductivity state [5]. Also nonmagnetic impurities causes pair breaking, since their potential, in general, does not transform in the same way as the order parameter [6]. There are also systems, such as UGe_2 and superfluid ^{3}He, in which the superconductivity and superfluidity are mediated by interactions with impurities [7]. In case of ^{3}He a direct contamination of this substance with any atomic impurities is impossible. Instead of this experimentalists use liquid ^{3}He to fill up aerogel that is a matrix of randomly arranged silica filaments of nanometer diameter [8]. There is also suspicion that impurities are crucial for high-temperature superconductivity.

In this paper, we consider topological defect production (as example of kinks) in the presence of spatial inhomogeneity, such as impurities, admixtures, and even crystalline net. The paper is organized as follows. The Halperin formula is generalized in the follwing section. Section III contains a description of the defect production in the presence of a single impurity. In Sec. IV, we generalize results of Sec. III to describe (in adiabatic approximation) a system with arbitrary spatial inhomogeneity. The final section lists remarks.

II. GENERALIZATION OF THE HALPERIN FORMULA

The number density of zeros of the scalar field can be calculated as a sum over all points x_i, defined by the equation $\phi(t,x_i)=0$, and located in the vicinity of the point x,

$$n(t,x)=\lim_{L\to 0}\frac{\langle N\rangle}{2L}=\lim_{L\to 0}\frac{1}{2L}\left\langle \sum_i \frac{|\phi'(t,x_i)|}{|\phi'(t,x_i)|}\right\rangle. \tag{1}$$

If we identify ϕ' with f and ϕ with g, then the lemma 1 allows are to replace the sum over zeros of the scalar field by the integral over the interval located in the neighborhood of the point x,

$$n(x)=\lim_{L\to 0}\frac{1}{2L}\left\langle \int_{x-L}^{x+L} d\tilde{x}\,|\phi'(t,\tilde{x})|\,\delta(\phi(t,\tilde{x}))\right\rangle. \tag{2}$$

In the zero L limit, this integral simplifies to the form

$$n(x)=\langle \mathrm{sgn}[\phi'(t,x)]\phi'(t,x)\,\delta(\phi(t,x))\rangle, \tag{3}$$

where we replaced a modulus by the product of the derivative of the scalar field ϕ' and its sign, i.e., $|\phi'(t,x)| =\mathrm{sgn}[\phi'(t,x)]\phi'(t,x)$. The integral representation of the delta function $\delta(\phi(t,x))=(1/2\pi)\int_{-\infty}^{\infty}ds\,e^{is\phi(t,x)}$ and the step function $\mathrm{sgn}[\phi(t,x)]=\int_{-\infty}^{\infty}(dz/\pi i)[e^{iz\phi'(t,x)}/z]$ allows for reformulation of the last formula to the more convenient form

$$n(x)=\frac{1}{2\pi^2 i}\left\langle \int_{-\infty}^{\infty}ds\int_{-\infty}^{\infty}\frac{dz}{z}\exp[iz\phi'(t,x)+is\phi(t,x)]\phi'(t,x)\right\rangle. \tag{4}$$

In the example considered here, the scalar field is a real order parameter of the one-dimensional ϕ^4 model

$$\partial_t\phi(t,x)=\partial_x^2\phi(t,x)-a(t)\phi(t,x)-\lambda\phi^3(t,x)+\eta(t,x)+\mathcal{D}(t,x), \tag{5}$$

where $\eta(t,x)$ can be a temperature Gaussian white noise or even a spatially correlated isotropic noise. A quantity $\mathcal{D}(t,x)$ is a deterministic force representing the existence of impurities or the crystalline net in the substance.

For the time sufficiently close to the instant of transition, ϕ is so small compared to the vacuum value that cubic term is negligible and dynamics is governed only by the linear terms. We assume that in a Gaussian approximation the influence of the thermal and deterministic forces on the order parameter is easily distinguished, i.e., $\phi(t,x)=\psi(t,x)+u(t,x)$, where $\psi(t,x)$ describes part of the evolution caused by a thermal noise and $u(t,x)$ is generated by the deterministic potential

$$\partial_t \psi(t,x)=\partial_x^2 \psi(t,x)-a(t)\psi(t,x)+\eta(t,x), \tag{6}$$

$$\partial_t u(t,x)=\partial_x^2 u(t,x)-a(t)u(t,x)+\mathcal{D}(t,x). \tag{7}$$

The number density of zeros of the scalar field under above assumptions splits into two parts as

$$n(x)=\frac{1}{2\pi^2 i}\int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty}\frac{dz}{z}e^{isu+izu'}\langle \psi' e^{is\psi+iz\psi'}\rangle + \frac{1}{2\pi^2 i}u'\int_{-\infty}^{\infty} ds\int_{-\infty}^{\infty}\frac{dz}{z}e^{isu+izu'}\langle e^{is\psi+iz\psi'}\rangle. \tag{8}$$

The significant progress in our calculation can be made with the help of the Lemmas 3 and 4,

$$n(x)=\frac{1}{2\pi}\langle \psi'^2\rangle \int_{-\infty}^{\infty} ds \exp\left[isu-\frac{1}{2}s^2\langle\psi^2\rangle\right]\int_{-\infty}^{\infty}dz \times\exp\left[izu'-\frac{1}{2}z^2\langle\psi'^2\rangle\right]+\frac{1}{2\pi^2 i}u'\int_{-\infty}^{\infty}ds \times\exp\left[isu-\frac{1}{2}s^2\langle\psi^2\rangle\right]\int_{-\infty}^{\infty}\frac{dz}{z} \times\exp\left[izu'-\frac{1}{2}z^2\langle\psi'^2\rangle\right]. \tag{9}$$

Three of the four integrals are of Gaussian type $\int_{-\infty}^{\infty}ds \exp[isu-\frac{1}{2}s^2\langle\psi^2\rangle]=\sqrt{2\pi/\langle\psi^2\rangle}e^{-u^2/2\langle\psi^2\rangle}$, therefore

$$n(x)=\frac{1}{\pi}\sqrt{\frac{\langle\psi'^2\rangle}{\langle\psi^2\rangle}}e^{-u^2/2\langle\psi^2\rangle}\left[e^{-u'^2/2\langle\psi'^2\rangle}+\frac{u'}{\sqrt{2\pi\langle\psi'^2\rangle}} \times\int_{-\infty}^{\infty}\frac{dz}{\pi i}\exp\left(izu'-\frac{1}{2}z^2\langle\psi'^2\rangle\right)\right]. \tag{10}$$

The last integral can be expressed via the error function (see the end of the Appendix)

$$n(t,x)=\frac{1}{\pi}\sqrt{\frac{\langle\psi'^2\rangle}{\langle\psi^2\rangle}}\exp\left[-\frac{u^2}{2\langle\psi^2\rangle}-\frac{u'^2}{2\langle\psi'^2\rangle}\right] + \frac{u'}{\sqrt{2\pi\langle\psi^2\rangle}}e^{-u^2/2\langle\psi^2\rangle}\,\mathrm{Erf}\left(\frac{u'}{\sqrt{2\langle\psi'^2\rangle}}\right). \tag{11}$$

In the case of $\mathcal{D}(t,x)=0$, $u(t,x)=0$ this formula reduces to the well-known Liu-Mazenko-Halperin formula [9]

$$n(t,x)=\frac{1}{\pi}\sqrt{\frac{\langle\psi'^2\rangle}{\langle\psi^2\rangle}}. \tag{12}$$

III. DEFECT PRODUCTION IN THE PRESENCE OF A SINGLE IMPURITY

For simplicity we assume an instantaneous quench, i.e., $a(t)=1$ for $t<0$ and $a(t)=-1$ for $t>0$. In this situation, time dependence of the chemical potential can be a consequence of the change of an external pressure. Fourier transformation $\psi(t,x)=\int_{-\infty}^{\infty}dk e^{ikx}\tilde{\psi}(t,k)$, $u(t,x)=\int_{-\infty}^{\infty}dk e^{ikx}\tilde{u}(t,k)$ allows for significant simplification of the equations of motion (6) and (7)

$$\partial_t\tilde{\psi}(t,k)+k^2\tilde{\psi}(t,k)+a(t)\tilde{\psi}(t,k)=\tilde{\eta}(t,k), \tag{13}$$

$$\partial_t\tilde{u}(t,k)+k^2\tilde{u}(t,k)+a(t)\tilde{u}(t,k)=\tilde{\mathcal{D}}(t,k). \tag{14}$$

The general solution of Eq. (13),

$$\tilde{\psi}(t,k)=\int_{-\infty}^{t}dt_1 \exp\left\{-\int_{t_1}^{t}dt_2[k^2+a(t_2)]\right\}\tilde{\eta}(t_1,k), \tag{15}$$

together with the white Gaussian noise cumulants

$$\langle\tilde{\eta}(t,k)\rangle=0,$$

$$\langle\tilde{\eta}^*(t,k)\tilde{\eta}(t',k')\rangle=\frac{T}{\pi}\delta(k-k')\delta(t-t'), \tag{16}$$

provide equal time correlators

$$\langle\psi^2\rangle=\frac{1}{2}T[(1-\mathrm{Erf}\sqrt{2t})e^{4t}+\mathrm{Erfi}(\sqrt{2t})], \tag{17}$$

$$\langle\psi'^2\rangle=\frac{1}{2}T\left[\frac{2}{\pi}k_m-(1-\mathrm{Erf}\sqrt{2t})e^{4t}+\mathrm{Erfi}(\sqrt{2t})\right], \tag{18}$$

where Erf and Erfi are, respectively, the error and inverse (blowing) error functions. Cutoff k_m in a momentum is inevitable because it removes an ultraviolet momentum divergence, which is caused by a large number of zeros of the field configuration provided by thermal fluctuations on small distances. Typical choice of k_m is an inverse of the correlation length $k_m=1/\xi$.

Let us find the solution of the Eq. (14) in the typical time-independent, coming from impurity, force $\mathcal{D}(x)=\mathcal{A}(x-x_0)e^{-(x-x_0)^2/\alpha^2}$,

$$u(t,x)=\tfrac{1}{8}\mathcal{A}\alpha^3 z[\exp(2t+\tfrac{1}{4}\alpha^2)\mathcal{I}_1+e^{-(1/4)\alpha^2}\mathcal{J}_1], \tag{19}$$

$$u'(t,x)=\tfrac{1}{8}\mathcal{A}\alpha^3[\exp(2t+\tfrac{1}{4}\alpha^2)(\mathcal{I}_1-\tfrac{1}{2}z^2\mathcal{I}_2) + e^{-(1/4)\alpha^2}(\mathcal{J}_1-\tfrac{1}{2}z^2\mathcal{J}_2)], \tag{20}$$

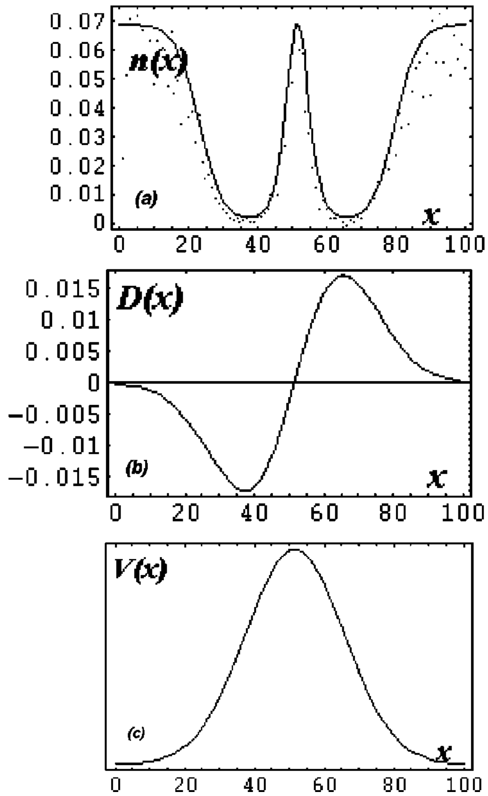

FIG. 1. Kinks are produced mainly in areas where deterministic force disappears. Parameters chosen in this plot are the following: $T=0.001$, $\mathcal{A}=0.002$, $\alpha=20$, $x_0=50$, $t=5.5$, and $k_m=1$. (a) The spatial kink distribution $n(x)$. (b) The deterministic force of the impurity $\mathcal{D}(x)$. (c) The impurity potential $V(x)$.

where $z=x-x_0$ is relative localization of the potential,

$$\mathcal{I}_n=\int_{t+(1/4)\alpha^2}^{\infty} ds \frac{e^{-s-(z^2/4s)}}{s^n\sqrt{s}}$$

and

$$\mathcal{J}_n=\int_{(1/4)\alpha^2}^{t+(1/4)\alpha^2} ds \frac{e^{-s-(z^2/4s)}}{s^n\sqrt{s}}.$$

If we assume slow x dependence of the deterministic force $\mathcal{D}(x)$, then Eqs. (19) and (20) can be approximated by the formulas

$$u(t,x)\approx \mathcal{A}ze^{-z^2/\alpha^2}e^t(2-e^{-t}), \tag{21}$$

$$u'(t,x)\approx \mathcal{A}\left(1-\frac{2z^2}{\alpha^2}\right)e^{-z^2/\alpha^2}e^t(2-e^{-t}). \tag{22}$$

The density of zeros of the Higgs field is given by the generalized Halperin formula (11).

The kink distribution and considered impurity potential are presented in Fig. 1.

The characteristic feature of the influence of the inhomogenous potential is expelling defects out of the regions where the nonzero deterministic force is present. In the regions where the force $\mathcal{D}(x)$ disappears, kinks are produced as a characteristic of the homogenous case number. This result agrees with conclusions of the papers [10] where a decrease in the number of produced defects in the presence of a constant and homogenous external field is predicted. Depending on the parameters of the potential the effect of the impurity on defect distribution can be significant or almost unobservable (see Fig. 2). Note that the formalism presented here can also be applied to description of more complicated disturbances, e.g., shock wave [11].

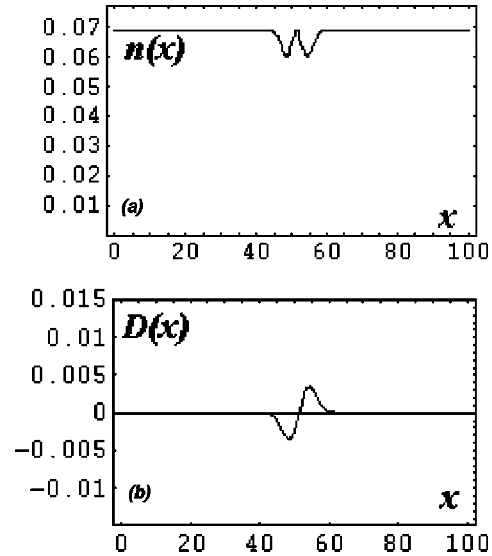

FIG. 2. For the weak impurity its influence on defect production can be almost unobservable. Parameters chosen in this plot are the following: $T=0.001$, $\mathcal{A}=0.002$, $\alpha=4$, $x_0=50$, $t=5.5$, and $k_m=1$. (a) The spatial distribution of produced kinks $n(x)$. (b) The deterministic force of the impurity $\mathcal{D}(x)$.

IV. AN INFLUENCE OF THE ARBITRARY TIME INDEPENDENT FORCE ON KINK PRODUCTION

If we consider the slow varying impurity potential then the first term on the right-hand side of Eq. (7) is unimportant,

$$\partial_t u(t,x)+a(t)u(t,x)=\mathcal{D}(t,x). \tag{23}$$

The Fourier transformation of this equation,

$$\partial_t \tilde{u}(t,k)+a(t)\tilde{u}(t,k)=\tilde{\mathcal{D}}(t,k), \tag{24}$$

under assumption of the time independence of the inhomogeneity force $\mathcal{D}=\mathcal{D}(x)$, leads to the solution $\tilde{u}(t,k)=\tilde{\mathcal{D}}(k)(2e^t-1)$. In space coordinates this solution has the form

$$u(t,x)=\mathcal{D}(x)(2e^t-1). \tag{25}$$

One could easy check that this solution coincides with the solution (21) for impurity force of the form $\mathcal{D}(x)=\mathcal{A}(x-x_0)e^{-(x-x_0)^2/\alpha^2}$ considered in the preceding section. The other representative force is a Gaussian type $\mathcal{D}(x)=\mathcal{A}e^{-(x-x_0)^2/\alpha^2}$, the potential for this force represents a kind of a junction of two mediums. In this case the defects are expelled from the junction (see Fig. 3). The Fig. 3 show

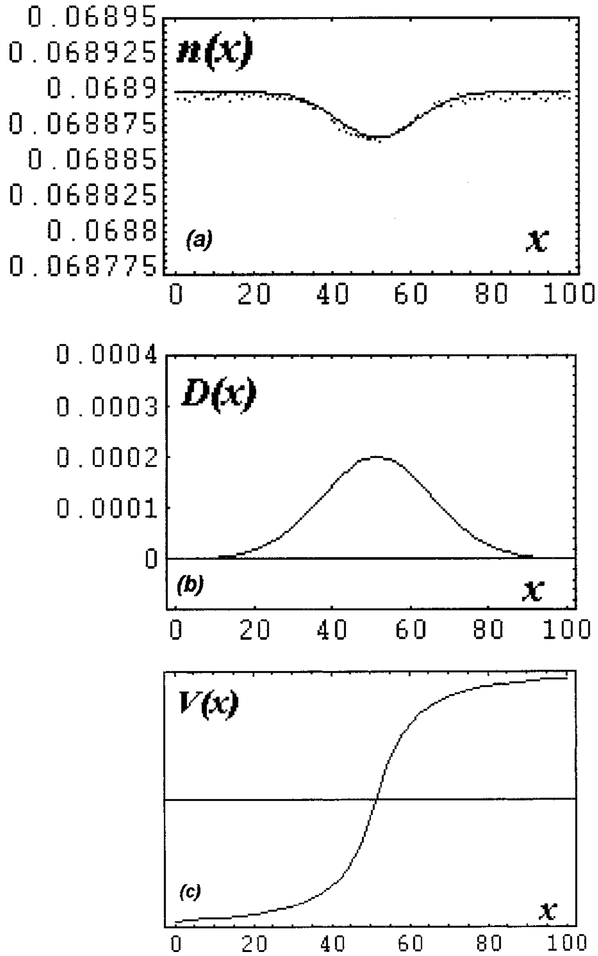

FIG. 3. Behavior of the system in the presence of Gaussian-type force $\mathcal{D}(x)$. The parameters are identical to those chosen in Fig. 1. $\mathcal{A}=0.002$, $\alpha=20$, $x_0=50$. (a) The kink distribution in the neighborhood of the junction. (b) The force in case of the junction. (c) The potential of the junction.

expelling kinks by the junction. In fact, Figs. 1 and 3 presents all generic features of the kink production in the presence of arbitrary inhomogenous medium. Actually, defects are expelled from regions where a nonzero force is present (Fig. 3) and trapped by knots of the function $\mathcal{D}(x)$—see Fig. 1. This general behavior of defects is not changed by the dimensionality of the system. If defects are produced in the crystalline medium, then the produced defects concentrate near the knots of the function $\mathcal{D}(x)$. For instance, for sinusoidal function $\mathcal{D}(x)=\mathcal{A}\sin[(x-x_0)/\alpha]$, kinks form a regular structure that has period 2 time smaller then the original net.

V. REMARKS

The impurity disturbs the system only locally [12]. The main feature of the influence of the inhomogenous potential is to expel defects out of the regions occupied by the impurity. Actually, kinks are expelled from the regions where the deterministic force $\mathcal{D}(x)$ coming from the impurity is different from zero. In the regions where the force disappears kinks are produced as a characteristic of the homogenous case number. Depending on the parameters of the potential the effect of the impurity on defect distribution can be significant or almost unobservable. In case of the junction, the defects are expelled from the area of contact of two mediums. The generic features of the kink production in the presence of an arbitrary inhomogenous medium is trapping defects by knots of the function $\mathcal{D}(x)$. This general behavior concerns also production of kinks in the crystalline medium where the defects concentrate near the knots. It seems that this feature of the defect production does not depend on the number of dimensions. Let us stress that results obtained here are superb starting point for generalization of this formalism to higher number of dimensions and gauge symmetric models.

APPENDIX

This section contains collection of the main results used in the proof of the Liu-Mazenko-Halperin formula.

Lemma 1. If x_i denotes the positions where $g(x_i)=0$ then

$$\int dx f(x)\,\delta(g(x))=\sum_i \frac{f(x_i)}{|g'(x_i)|}.$$

Lemma 2. If we consider a spatially correlated noise, i.e., if

$$\langle\tilde{\eta}(t,k)\rangle=0 \quad \text{and} \quad \langle\tilde{\eta}(t,k)\tilde{\eta}(t,k')\rangle=\tilde{f}(k^2)\,\delta(k-k'),$$

then

$$\langle\psi(t,x)\rangle=0,\quad \langle\psi'(t,x)\rangle=0,\quad \langle\psi(t,x)\psi'(t,x)\rangle=0.$$

The proof of this lemma is immediate consequence of the Fourier transformation of the solution given by Eq. (15).

Lemma 3. If

$$\langle\tilde{\eta}(t,k)\rangle=0 \quad \text{and} \quad \langle\tilde{\eta}(t,k)\tilde{\eta}(t,k')\rangle=\tilde{f}(k^2)\,\delta(k-k')\rangle,$$

then

$$\langle\psi^{2n}\rangle=(2n-1)!!\langle\psi^2\rangle^n,\quad \langle\psi'^{2n}\rangle=(2n-1)!!\langle\psi'^2\rangle^n,$$

$$\langle\psi^{2n}\psi'^{2k}\rangle=(2n-1)!!(2k-1)!!\langle\psi^2\rangle^n\langle\psi'^2\rangle^k,$$

$$\langle\psi^{2n}\psi'^{2k+1}\rangle=\langle\psi^{2n+1}\psi'^{2k}\rangle=\langle\psi^{2n+1}\psi'^{2k+1}\rangle=0.$$

This lemma is a consequence of the Wick theorem.

Lemma 4.

$$\langle e^{is\psi}\rangle=e^{-(1/2)s^2\langle\psi^2\rangle}.$$

According to Lemma 3, odd terms in the expansion of the left-hand side of the above equation are absent and, therefore,

$$\langle e^{is\psi}\rangle=\sum_{n=0}^{\infty}\frac{(-1)^n(s^2)^n\langle\psi^{2n}\rangle}{(2n)!}.$$

If we use identity $(2n-1)!!=(2n)!/2^n n!$ then the coefficients of the expansion can be transformed with the use of the lemma 3 to the form $[1/(2n)!](-1)^n(s^2)^n\langle\psi^{2n}\rangle=(1/2^n)[1/(n)!](-1)^n(s^2)^n\langle\psi^2\rangle^n$. As a result of summing up of those coefficients we obtain the right-hand side of Lemma 4.

Lemma 5.

$$\langle e^{is\psi+iz\psi'}\rangle=\exp\left[-\frac{1}{2}s^2\langle\psi^2\rangle\right]\exp\left[-\frac{1}{2}z^2\langle\psi'^2\rangle\right].$$

The left-hand side of the lemma can be expanded as

$$\langle e^{is\psi+iz\psi'}\rangle=\sum_{k=0}^{\infty}\frac{1}{k!}\sum_{j=0}^{k}\binom{k}{j}(is)^{k-j}(iz)^j\langle\psi^{k-j}\psi'^j\rangle,$$

and then reformulated with the use of Lemma 3:

$$\langle e^{is\psi+iz\psi'}\rangle=\sum_{n=0}^{\infty}\sum_{r=0}^{n}\frac{1}{(n-r)!r!}\left(-\frac{1}{2}s^2\langle\psi^2\rangle\right)^{n-r}\times\left(-\frac{1}{2}z^2\langle\psi'^2\rangle\right)^r.$$

After renumbering of the series

$$\sum_{n=0}^{\infty}\sum_{r=0}^{n}a_{n-r}b_r=\sum_{n=0}^{\infty}a_n\sum_{r=0}^{\infty}b_r,$$

we obtain the right-hand side of Lemma 5.

Lemma 6.

$$\langle\psi' e^{is\psi+iz\psi'}\rangle=iz\langle\psi'^2\rangle\exp\left[-\frac{1}{2}s^2\langle\psi^2\rangle\right]\exp\left[-\frac{1}{2}z^2\langle\psi'^2\rangle\right].$$

Lemmas 3 and 4 provide $\langle e^{is\psi+iz\psi'}\rangle=\langle e^{is\psi}\rangle\langle e^{iz\psi'}\rangle$. In the same way we prove $\langle\psi' e^{is\psi+iz\psi'}\rangle=\langle e^{is\psi}\rangle\langle\psi' e^{iz\psi'}\rangle$. The second term of this product with the help of Lemma 3 transforms as follows:

$$\langle\psi' e^{iz\psi'}\rangle=-i\frac{\partial}{\partial z}\langle e^{iz\psi'}\rangle=-i\frac{\partial}{\partial z}\exp\left[-\frac{1}{2}z^2\langle\psi'^2\rangle\right].$$

Explicit differentiation of the last term gives the proper factor on the right-hand side of Lemma 6.

In the end of this section, we evaluate the last integral in the formula (10). Let us denote

$$\mathrm{P}\int_{-\infty}^{\infty}\frac{dz}{zi}\exp\left[izu'-\frac{1}{2}z^2\langle\psi'^2\rangle\right]\equiv g(u'),$$

then by differentiation with respect to u' we obtain the Gaussian integral

$$\frac{dg}{du'}=\sqrt{\frac{2\pi}{\langle\psi'^2\rangle}}e^{-u'^2/2\langle\psi'^2\rangle}.$$

Finally, after a second integration, a function $g(u')$ appears to be proportional to the error function,

$$g(u')=\sqrt{\frac{2\pi}{\langle\psi'^2\rangle}}\int_0^{u'}d\tilde{u}'e^{-\tilde{u}'^2/2\langle\psi'^2\rangle}.$$

[1] A.R. Bishop, J.A. Krumhansl, and S.E. Trullinger, Physica D **1**, 44 (1980).

[2] V.A. Kuzmin, V.A. Rubakov, and M.S. Shaposhnikov, Phys. Lett. **155B**, 36 (1985).

[3] M. Peyrard and A.R. Bishop, Phys. Rev. Lett. **62**, 2755 (1989).

[4] W.H. Zurek, Nature (London) **317**, 505 (1993); Acta Phys. Pol. B **24**, 1301 (1993); Phys. Rep. **276**, 177 (1996).

[5] K. Maki, in Superconductivity, edited by R. Parks, Pt. II, p. 1035 (unpublished).

[6] G. Har'an and A. Nagi, Phys. Rev. B **58**, 12 441 (1998).

[7] E.W. Hudson, K.M. Lang, V. Madhavan, S.H. Pan, H. Eisaki, S. Uchida, and J.C. Davis, Nature (London) **411**, 920 (2001).

[8] G. Lawes, S.C.J. Kingsley, N. Mulders, and J.M. Parpia, Phys. Rev. Lett. **84**, 4148 (2000).

[9] B. Halperin, in *Physics of Defects*, edited by R. Balian, M. Kleman, and J.P. Poirier (North-Holland, New York, 1981); A. Weinrib and B.I. Halperin, Phys. Rev. B **26**, 1362 (1982); F. Liu and G.F. Mazenko, *ibid.* **46**, 5963 (1992).

[10] J. Dziarmaga, Phys. Rev. Lett. **81**, 5485 (1998); J. Dziarmaga and M. Sadzikowski, Phys. Rev. Lett. **82**, 4192 (1999).

[11] J. Dziarmaga, P. Laguna, and W.H. Zurek, Phys. Rev. Lett. **82**, 4749 (1999).

[12] K. Ishida, Y. Kitaoka, K. Yamazoe, K. Asayama, and Y. Yamada, Phys. Rev. Lett. **76**, 531 (1996).